Math 131A-3: Homework 3

Due: October 18, 2013

1. Do problems 8.5, 8.9, 9.3, 9.12, 9.14, 10.6, 10.7, 10.10 in Ross. [You may assume the result of 9.13 in writing up 9.14, but understanding this exercise on your own time is also recommended.]

2. We say a subset S of $\mathbb{R}$ is *closed* if whenever a sequence (s_n) of numbers in S converges to some s, the limit s is also in S.

   (a) Use exercise (8.9) above to show that that the interval $[a, b]$ is closed for any real $a < b$.

   (b) Give an example of a closed unbounded set in $\mathbb{R}$.

   (c) Suppose $S \subset \mathbb{R}$ is closed and bounded above. Use exercise (10.7) above to show that S has a maximum.

3. Let (s_n) be the sequence $s_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$.

   (a) Show that s_n is increasing and bounded above by 2. [Hint: Multiply s_n by 2^n, then use an induction formula for $2^n + 2^{n-1} + \cdots 2^2 + 2 + 1$ proved in the first lecture. Later in the course we will see a general formula for this sort of sum.]

   (b) Show that $s_{n+1} = \frac{1}{2}s_n + 1$, and conclude that $\lim s_n = 2$.

   (c) Let $t_n = 1 + 2 + 4 + \cdots 2^n$. Observe that $t_{n+1} = 2t_n + 1$. Why can't we conclude that $\lim t_n = -1$?

   Note that this addresses one of our motivational questions from the first lecture!